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Battery testing by calculated discharge-curve method—Constant resistive load algorithm

Aleksandar B. Djordjevic [*], Dusan M. Karanovic

Mirijevski Bulevar 48, 11060 Belgrade, Serbia

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Abstract

Battery testing by calculated discharge-curve method (CDCM) includes battery, battery average cell and each of battery's cell 3D discharge curves. These discharge curves were generated by calculated discharge-curve algorithm (CDCA), using either high- or low-frequency monitored discharge data and are lying in battery/cell rectangular parallelepiped: open-circuit voltage–initial discharge current–time.

CDCA defines the ordered six-tuple: time–current–voltage–capacity–energy on load–internal energy losses by which battery and battery's cells dynamic characteristics may be analyzed. Any of the six-tuple variables may be divided into the set of $i = 1, \ldots, n$ steps by constant step across the overall discharge interval and used as the domain of CDC algorithm procedure.

CDCM, respectively, introduced average current and average voltage as the ratio of capacity to time and energy to capacity. Battery/cell rectangular parallelepiped: time–average current–average voltage is defined at any step of discharging.

In this paper, the linear dependencies of average characteristics on the both power of battery/cell internal resistance and load intensity were presented and algebrically proofed. Battery's cells may be compared to battery average cell and may be classified by any characteristics.

Discharge curves of alkaline-manganese MALLORY batteries, 9K62 (50 and 100 Ω), PX24 (100 and 166 Ω), PX21 (150 and 250 Ω) and 7K62 (100, 200 and 500 Ω) [T.R. Crompton, Small Batteries 2 (1982) 52] were used to demonstrate battery and battery average cell standard and CDC method characteristics.

The mathematical calculations were conducting on IBM PC using Microsoft Excel software.

Keywords: Galvanic battery; Battery average cell; Battery's cells classification

1. Introduction

Cell testing by calculated discharge-curve method (CDCM) has been described [3]. In this article, battery testing by CDCM was presented.

The term battery defines a number of cells arranged in series or parallel. The cells are of the same chemical couple, cell shape and size as well as the same cell design and manufacturing method. Battery of particular cell is the smallest unit or section, which are fitted with electrical terminals. A number of cells arranged in series give a battery voltage as the sum of the battery's cells' voltages. Battery of average cell may represent the battery:

$$V_{\text{average cell},ii} = \frac{V_{\text{battery},i}}{N} \tag{1}$$

where N denotes the number of cells. A good battery consists of the cells of the same quality, which means the same cell's dynamic characteristics can be expected under the discharge conditions. If the dynamic characteristics of battery average cell and every battery's single cell were plotted versus the same voltage interval, every particular cell may be compared to the battery average cell and battery cell classification may be performed. A battery characteristic load, temperature dependence, secondary battery standby exploita-

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[*] Corresponding author. Tel.: +381 011 784 547.
E-mail address: aldjordj@EUnet.yu (A.B. Djordjevic).

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tion history and cycle life may be performed and analyzed. In addition, there is a possibility to plot characteristics of the batteries of the same chemical couple, but various shapes, size, designs and manufacturing method versus the same voltage interval.

1.1. Calculated discharge-curve algorithm

Calculated discharge-curve algorithm (CDCA) is the mathematical tool by which battery/cell discharge curves were generated [3] and their CDCM analysis may be performed.

CDCA defines the ordered six-tuple:

Time (s)	$-t_0 < t_i < t_n$
Current (A)	$I_0 > I_i > I_n$
Voltage (V)	$V_0 > V_i > V_n$
Capacity/As	$Q_0 < Q_i < Q_n$
Energy on load (VAs)	$E_0 < E_i < E_n$
Energy losses (VAs)	$E_{\text{int},0} < E_{\text{int},i} < E_{\text{int},n}$

which are associated with the set of ordered number: $0 \leq i \leq n$. 3D-space: time–current–voltage was chosen to put six-tuple together. Instead of 3D graph t (s)–I (A)–V (V) the projections of 3D discharge curve [3] may be drawn on the every coordinate planes: time–current, time–voltage and current–voltage.

1.2. Energy volume

One can observe that sum of the energy on load and overall energy losses are the theoretical available battery energy [2], the volume of which has the extent: $I_0 \times U_0 \times t$ and the figure is a regular parallelepiped. The base of the parallelepiped is the product $I_0 \times U_0$ and height is the actual discharge time as an external variable. During discharge with no energy losses, the voltage, U_0 and current, I_0 remain constant and discharge time increases height of the four prism's sides, defining the sum of the all energies delivered.

The practical available energy (energy on load, E_i (VAs) is smaller than the theoretical one due to the decreasing of the both battery current and voltage during either self-discharge or current–voltage–power controlled discharge. The figure of the energy on load volume may be described as an irregular parallelepiped by the two pair of sides: (1) time–voltage surfaces at $I = 0$ and $I_0 > I_t \geq I_0/2$ and (2) two time–current at $V = 0$ and $U_0 > V_t \geq U_0/2$. The departure of the both current, $I_{t\text{ and}}$ voltage, V_t from the initial values defines 3D particular cell discharge curve.

The battery/cell overall energy losses, E_{overall} may be integrated across time scale:

$$E_{\text{overall},i} = \sum_i P_{\text{overall},i} \Delta t_i = \sum_i (U_0 I_0 - V_i I_{\text{battery},i}) \Delta t_i \tag{2}$$

where voltages, U_0 and V denote either battery or cell open-circuit and discharge voltage, respectively.

CDCA recognizes [3] battery/cell internal energy losses [1], E_{int} that may be integrated across time scale

$$E_{\text{int},i} = \sum_i P_{\text{o},i} \Delta t_i = \sum_i (U_0 - V_i) I_{\text{battery},i} \Delta t_i \tag{3}$$

where P_{o}/VA is battery/cell power of internal resistance [1]

$$P_{\text{o},i} = (U_0 - V_i) I_{\text{battery},i} \tag{4}$$

where voltages, U_0 and V denote either battery or cell open-circuit and discharge voltage, respectively.

1.3. Discharge-curve generation

The term discharge curve covers the family of the chosen ordered pairs, which represents battery/cell discharge characteristics. The evolution of CDC algorithm has been presented ([3], p.137) and we recall Eq. (19) into three forms:

$$\frac{\ln(y_i/y_{i-1})}{\ln(y_{i-1}/y_{i-2})} = \frac{\ln(P_{\text{o},i}/P_{\text{o},i-1})}{\ln(P_{\text{o},i-1}/P_{\text{o},i-2})} \tag{5a}$$

or

$$\frac{\Delta y_i/y_{i-1}}{\Delta y_{i-1}/y_{i-2}} = \frac{\Delta P_{\text{o},i}/P_{\text{o},i-1}}{\Delta P_{\text{o},i-1}/P_{\text{o},i-2}} \tag{5b}$$

or

$$f(y_i) = g(P_{\text{o},i}) \tag{5c}$$

where 'y_i' denotes discharge time or any extensive battery/cell characteristic, i.e. capacity, energy on load and internal energy losses. The left-hand expressions of Eq. (5c) was named y-operator, $f(y_i)$ and the right-hand expressions voltage operator, $g\,(P_{\text{o},i})$. If the five variables of the six members in Eq. (5) are known the sixth one may be calculated [3].

1.4. Average discharge characteristics

CDC algorithm calculates battery/cell average characteristics at each step, $0 \leq i \leq n$, of discharging. In this paper, battery/cell average characteristics were introduced by the following equations:

$$Q_i = \left(\frac{Q_i}{t_i}\right) t_i = (I_{\text{avg},i})\,\text{constant} \times t_i \tag{6}$$

$$E_i = \left(\frac{E_i}{Q_i}\right) Q_i = (V_{\text{avg},i})\,\text{constant} \times Q_i \tag{7}$$

$$E_i = \left(\frac{E_i}{t_i}\right) t_i = [(VI)_{\text{avg},i}]\,\text{constant} \times t_i \tag{8}$$

$$E_{\text{int},i} = \left(\frac{E_{\text{int}}}{t_i}\right) t_i = (P_{\text{avg},i})\,\text{constant} \times t_i \tag{9}$$

In accordance to the equations above cell/battery discharge may be performed by controlled discharge parameter and either using passive (auto-regulated resistive load) or active load (galvanostat, potentiostat or programmable electronic load).

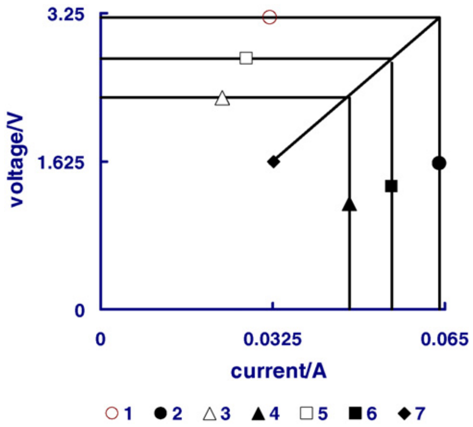

Fig. 1. Power plane, voltage vs. current of the co-ordinate system: time–current–voltage at $t = 18437$ s, $I = 0.047$ A and $V = 2.320$ V, $E/Q = V_{avg} = 2.75$ V and $Q/t = I_{avg} = 0.064$ A. MALLORY battery 9K62, constant resistive load, 50 Ω. Symbols and lines—1: battery open-circuit voltage, U_0 (V); 2: battery ideal current, I_0 (A); 3: battery discharge voltage, V (V); 4: battery discharge current, I (A); 5: battery average voltage, $E/Q = V_{avg,i}$ (V); 6: battery average current, $Q/t = I_{avg}$ (A); 7: battery polarization curve: V vs. I and E/Q vs. Q/t.

1.5. Power plane of energy volume

The projection of 3D discharge curve, *t*–*I*–*V*, of MALLORY alkaline-manganese battery 9K62, load 50 Ω, at the arbitrary discharge time point, *t* (s) is shown in Fig. 1 (line 7) and identify as polarization curve. The smallest parallelogram, bonded by the abscissas $V = 0$ and $V_{t=18437\,\mathrm{s}} = 1.16$ V and by the ordinates $I = 0$ and $I_{t=18437\,\mathrm{s}} = 0.047$ A, has started from the very beginning of discharge and has moving along time-scale. The largest parallelogram, bonded by the abscissas $V = 0$ and U_0 and by ordinates $I = 0$ and I_0, remain constant.

The projection of 3D set, t–I_{avg}–V_{avg}, is added in this scatter diagram (line 7), i.e. overlapping the polarization curve. The parallelogram, bonded by the abscissas $V = 0$ and $V_{avg,t=18,437\,\mathrm{s}} = 2.75$ V and by the ordinates $I = 0$ and $I_{avg,t=18,437\,\mathrm{s}} = 0.055$ A, has started from the very beginning of discharge and has transforming energy on load volume from irregular to regular parallelepiped.

If at any point, *t*–*I*–*V*, of 3D discharge curve, the parameters, $I_{avg,t}$, $V_{avg,t}$ and $(IV)_{avg,t}$ will times with actual discharge time, the withdrawn capacity, Q_i/A_s, time–voltage surface, RlQ_i (V_s) and energy on load, E_i (VAs) will be calculated, respectively.

If an identical battery/cell should be discharged by constant current, voltage or power, the actual parameters, $I_{avg,t}$, $V_{avg,t}$ and $(IV)_{avg,t}$ need to be applied to achieve the equal capacity, time–voltage surface or energy delivered. However, one cannot expect the identical voltage profile, current profile or discharge duration by the both self-driving and controlled discharge. Since CDCA is valid for the every discharge mode (constant or time-dependent current–voltage, etc.) the power plane may be defined across the overall discharge interval. In the case of electronically controlled and time-dependent current or voltage, a curved polarization [2] curve should be appearing on the power plane.

2. Experimental

The published discharge data ([4], p. 52) of MALLORY alkaline-manganese batteries: 9K62, PX24, PX21 and 7k67, instead of our own experimentally registered data, should be used in this article.

The ordered pairs, time versus discharge voltage, were picked out from graphs across the overall discharge interval: $V_0 \geq V_i \geq V_{cutoff}$ (see Tables 1A–1D). The seven tabulated pairs, voltage versus time trace the discharge profile across the overall discharge interval. CDC algorithm generates the

Table 1C
Input data, battery PX21, three cells, $U_0 = 4.8\ \mathrm{V} > V_i \geq V_{cutoff}$ $V \geq 2.4$ V

Load (Ω)	Discharge time (s)						
	4.2 V	3.9 V	3.6 V	3.3 V	3.0 V	2.7 V	2.4 V
150	–	5143	14143	27000	45000	63000	72000
250	3857	19286	42429	72000	111000	141424	154280

Table 1A
Input data, battery 9K62, two cells, $U_0 = 3.2\ \mathrm{V} > V_i \geq V_{cutoff} \geq 1.6$ V

Load (Ω)	Discharge time (s)						
	2.8 V	2.6 V	2.4 V	2.2 V	2.0 V	1.8 V	1.6 V
50	4000	9900	17000	23400	27900	32000	35200
100	6400	17600	32800	48600	59400	66600	71200
250	12600	54000	114400	136000	152000	166500	179200

Table 1B
Input data, battery PX24, two cells, $U_0 = 3.2\ \mathrm{V} > V_i \geq V_{cutoff} \geq 1.6$ V

Load (Ω)	Discharge time (s)						
	2.8 V	2.6 V	2.4 V	2.2 V	2.0 V	1.8 V	1.6 V
100	–	5143	14143	27000	45000	63000	72000
166	3857	19286	42429	72000	111000	141424	154280

Table 1D
Input data, battery 7 K67, four cells, $U_0 = 6.4\text{ V} > V_i \geq V_{\text{cutoff}} \geq 3.2\text{ V}$

Load (Ω)	Discharge time (s)						
	5.6 V	5.2 V	4.8 V	4.4 V	4.0 V	3.6 V	3.2 V
100	4000	9900	17000	23400	27900	32000	35200
250	6400	17600	32800	48600	59400	66600	71200
500	12600	54000	114400	136000	152000	166500	179200

set of ordered pairs, time, t_i versus voltage, V_i, $0 \leq i \leq n$, $n = 320$, which overlap the input pairs data. The generated set of ordered pairs was accepted as the monitored discharge curve by the reading parameter which was used by CDC algorithm procedure, $\Delta y = \text{constant}$ (y: V, fV, t, Q, E) which was described earlier [3], i.e. uses Eq. (5a) in this paper.

3. Results and disussion

3.1. Discharge curve

CDCA is enable to generate discharge curves as ordered pairs, voltage versus (1) time, (2) capacity, (3) energy on load and (4) internal energy losses, dividing domain sets by constant step. In addition, CDCA evaluated the family of discharge curves: (1) time, (2) capacity, (3) energy on load and (4) internal energy losses versus the common voltage domain either battery $(U_0 > V_i \geq V_{\text{cutoff}})_{\text{battery}}$ or battery average cell $(U_0 > V_i \geq V_{\text{cutoff}})_{\text{battery average cell}}$.

The two scatter diagrams of the all batteries tested and load applied:

- discharge time versus battery average cell (Fig. 2);
- battery voltage versus time (Fig. 3).

They were presented in this paper due to the limited paper volume. Since the any discharge curve is a set of ordered pairs, the battery/cell capacity and energies should be represented by the equations.

3.1.1. Time–voltage curves

The set of ordered pairs, time, t_i versus voltage, V_i, $0 \leq i \leq n$, $n = 320$, using Eq. (5a) and by procedure described earlier [3], was generated by CDCA for the each of the battery discharge. Generated discharge curves were accepted as experimentally ones due to the fact that the linear regressions:

$$t_{\text{experimental}} = b \times t_{\text{generated}} \tag{10}$$

show the regression coefficients enough large: $R^2 > 0.999$. Ten discharge battery's average cell curves were generated for ten discharges of four battery types and graphically presented, Fig. 2. Using these discharge curves, 10 battery's discharge curves were calculated by Eq. (1) and put into scatter diagram (Fig. 3). Ten battery's average cell curves are grouped into five pairs, i.e. the curves of the same cell type and load are overlapped: 1 and 8, 2 and 9, 3 and 10, 4 and 6 and 5 and 7. The batteries 9K62 and 7K67 consist of two and four cells of the same type, as well as the batteries PX24 and PX21, two and three cells.

If a battery testing includes every cell voltage versus time reading, the family of battery average cell and every battery's cell need to be generated. Discharge curve of battery's particular cell may or not overlapped the curve of battery average cell. After the battery discharge test, only the worst cell was achieved the predetermined cutoff voltage, while the others battery's cells were not achieved the predetermined cutoff voltage. CDC algorithm generates cell discharge curve of the other battery's cells to the predetermined cutoff voltage and classified cells regarding discharge duration. Cell-generated discharge duration and battery-current profile

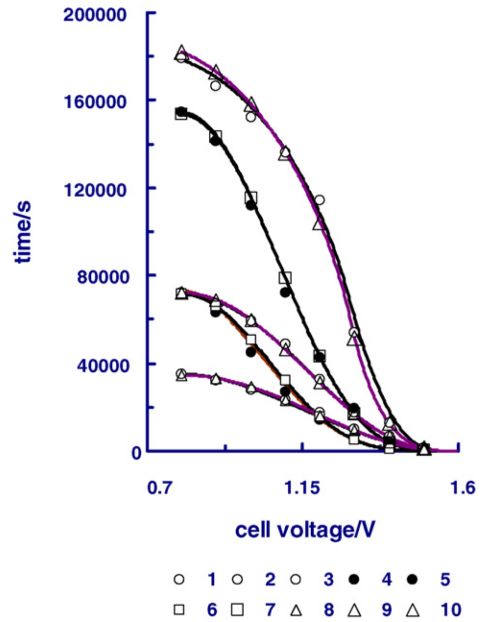

Fig. 2. Time vs. voltage of average cell, MALLORY battery, constant resistive load. Symbols—input discharge data (see Tables 1A–1D). Lines—generated sets of ordered pairs: t_i vs. V_i, $0 < i \leq n = 320$.

Battery	9K62 2 cells			PX24 2 cells		PX21 3 cells		7K67 4 cells		
Curve number	1	2	3	4	5	6	7	8	9	10
Load per cell (Ω)	25	50	125	50	83	50	83	25	50	125

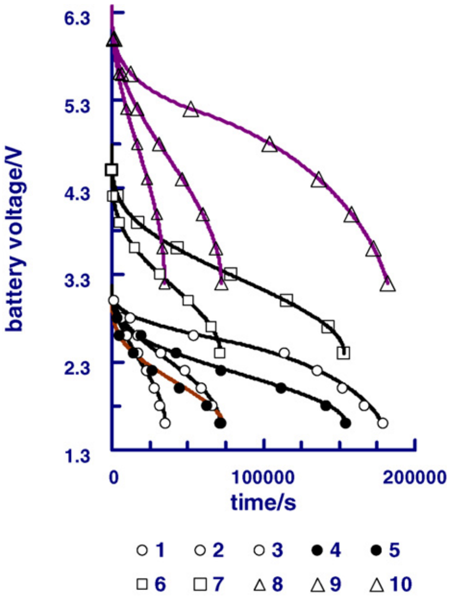

Fig. 3. Battery voltage vs. time, MALLORY battery, constant resistive load. Symbols—input discharge data (see Tables 1A–1D). Lines—generated sets of ordered pairs: t_i vs. V_i, $0<i\leq n=320$.

Battery	9K62 2 cells			PX24 2 cells		PX21 3 cells		7K67 4 cells		
Curve number	1	2	3	4	5	6	7	8	9	10
Load per cell (Ω)	50	100	250	100	166	150	250	100	200	500

defines cell's available capacity as the cell has been discharged separately. Cell discharge duration, battery-current profile and cell-voltage profile defines cell's available energy on load as the cell has been discharged separately. In conclusion, generated discharge duration, available capacity and energy delivered of each battery's cell as well as of the battery average cell may be used to perform battery's cells classification.

3.1.2. Capacity

The capacity per cell, actually withdrawn to the time point t_i, may be successfully calculated by:

$$Q_i=\sum_i I_i\Delta t_i=I_it_i+\sum_i\Delta I_it_i=I_0t_i-\sum_i(I_0-I_i)\Delta t_i$$
$$=(I_{\text{avg},i})\,\text{constant}\,t_i \tag{11}$$

The parallelogram $(I_0–I_i)t_i$ was divided by the time–current curve into the two similar halves, one of which represents current losses and the other represents the part of capacity surface: $\Sigma\Delta I_it_i=Q_i-I_it_i$. The product: $(I_{\text{avg},i})_{\text{constant}}\times t_i$ denotes that constant current discharge may be 'performed' instead of the constant resistive load discharge represented by the Eq. (11) at all.

3.1.3. Energy on load

The energy on load per cell, actually withdrawn to the time point t_i, may be successfully calculated by:

$$E_i=\sum_i I_i\Delta t_i=I_iV_it_i+\sum_i\Delta(IV)_it_i$$
$$=I_0\underset{-}{U}_0t_i-\sum_i(I_0\underset{-}{U}_0-I_iV_i)\Delta t_i=(IV)_{\text{avg},i}\text{constant}\,t_i \tag{12}$$

The first expression on the right-hand of Eq. (12) represents a cell energy volume as an irregular prism. The base of energy prism is the cell capacity area. The four sides are positioned on the borders of the capacity area. The outside top covering of the prism is the curved 3D-space surface (I–t–V) that depends on the cell discharge voltage profile. The second expression on the right-hand of Eq. (12) represents the cell energy volume as the sum of the parallelepiped ($I_iV_it_i$) and part above the parallelepiped. The second expression on the right-hand of Eq. (12) represents the cell energy as the quadrangular prism (base: $I_{\text{avg},i}\times V_{\text{avg},i}$ and height: t_i), which volume is equal to cell's energy volume. The product: $((IV)_{\text{avg},i})_{\text{constant}}t_i$ denotes that constant power discharge may be 'performed' instead of the constant resistive load discharge which are represented by the Eq. (12) at all. The irregular prism was transformed into the parallelepiped: $((IV)_{\text{avg},i})_{\text{constant}}t_i$. Instead of the constant power, either the constant current, $(I_{\text{avg},i})_{\text{constant}}$ or constant voltage, $(V_{\text{avg},i})_{\text{constant}}$ discharge

$$E_i=(I_{\text{avg},i})\text{constant}\sum V_i\Delta t_i=(V_{\text{avg},i})\text{constant}\sum I_i\Delta t_i \tag{13}$$

may be performed at any stage of discharge instead of the constant resistive load discharge, which are represented by the Eq. (12) at all.

3.1.4. Internal energy losses

The power of battery/cell internal resistance, i.e. the power of battery/cell polarization, may be expressed by rearranging Eq. (4) in this article:

$$P_{\text{o},i}=(U_0-V_i)I_{\text{battery},i}=(I_0-I_i)V_{\text{battery},i}V_i$$
$$=\left(U_0-\frac{\Delta E_i}{\Delta Q_i}\right)\frac{\Delta Qi}{\Delta t_i} \tag{14}$$

and

$$P_{\text{o},i}\Delta t_i=U_0\Delta Q_i-\Delta E_i \tag{15}$$

Battery/cell internal energy losses may be calculated by summation of Eq. (15):

$$E_{\text{int},i}=\sum_i P_{\text{o},i}\Delta t_i=U_0\sum_i\Delta Q_i-\sum_i\Delta E_i=U_0Q_i-E_i \tag{16}$$

where voltages, U_0 and V denote either battery or cell open-circuit and discharge voltage, respectively. Energy volume:

U_0Q_i is a prism which backward side is a curved surface. This energy volume is the sum of energy on load volume, E_i and internal energy losses one, $E_{\text{int},i}$. Divided Eq. (16) by time, we obtain:

$$\frac{E_{\text{int},i}}{t_i} = \frac{U_0Q_i}{t_i} - \frac{E_i}{t_i} = \left(U_0 - \frac{E_i}{Q_i}\right)\frac{Q_i}{t_i} = \left(I_0 - \frac{Q_i}{t_i}\right)\frac{E_i}{Q_i} \tag{17}$$

i.e.

$$P_{\text{avg},i} = U_0 I_{\text{avg},ii} - (IV)I_{\text{avg},i} = (U_0 - V_{\text{avg},i})I_{\text{avg},i} = (I_0 - I_{\text{avg},i})V_{\text{avg},i} \tag{18}$$

or

$$P_{\text{avg},i} = (U_0 - V_{\text{avg},i})\frac{V_{\text{avg},i}}{RI} = RI(I_0 - I_{\text{avg},i})V_{\text{avg},i} \tag{19}$$

Battery/cell average characteristics (I_{avg}, V_{avg}, $(IV)_{\text{avg}}$ and P_{avg}) linear relations to the battery/cell measurable characteristics (I, V and P_o) were investigated by algebraic way and checked by regression analysis.

The dependencies of battery average power, $P_{\text{avg},i}$ on the common domain, i.e.

$$P_{\text{battery,avg},i} = f[RI(U_0 - V_i)V_i]_{\text{average cell}} \tag{20}$$

The linear dependence of the 10 sets of battery average power, $P_{\text{avg},i}$ on the common domain was found and may be expressed by:

$$P_{\text{battery,avg},i} = b \times [(U_0 - V_i)V_i]_{\text{average cell}} \tag{21}$$

where 'b' denotes the slope of the straight line in coordinate system: $P_{\text{battery,avg}}$ versus $[(U_0 - V)V]_{\text{average cell}}$ (see Eqs., (17)–(19)).

The coefficient, b and the regression coefficient, R^2 of the 10 straight lines are:

Battery	Load (Ω)	Slope	Regression coefficient
9K62	50	0.0291	0.9996
	100	0.0149	0.9997
	250	0.0058	0.9796
PX24	100	0.0169	0.9998
	166	0.0099	0.9999
PX21	150	0.0253	0.9999
	250	0.0147	0.9999
7K67	100	0.0581	0.9999
	200	0.0299	0.9998
	500	0.0119	0.9838

The straight-line slopes relate to the load intensity, i.e. the linear dependence of the battery average characteristics on the load intensity (see Eqs. (17)–(20)) were defined.

3.2. Discharge-curve generation mode

Regarding the ordered six-tuple: time–current–voltage–capacity–energy on load–internal energy losses and Eq. (5) any of the six-tuple variables may be used as the domain while the all others as the range. In addition to the curve 2, Fig. 2 (t_i versus V_i), the new three discharge curves: $V_{t,i}$ versus t_i, $V_{Q,i}$ versus Q_i and $V_{E,i}$ versus E_i, $U_0 \geq V_i \geq U_0/2$ and $0 \leq i \leq n = 320$ were generated by: Δt, ΔQ and ΔE ordered sets. These four curves are plotted as scatter diagram in Fig. 4.

Table 2 is associated to avoid the numerical view in the generated successive steps. The various battery cell techniques may be analyzed by the one of the ordered pairs, voltage versus domain.

Table 2
Generated discharge ordered pairs battery 9K62, two cells, load 100 Ω, $U_0 = 3.2\ \text{V} > V_i \geq V_{\text{cutoff}} \geq 1.6\ \text{V}$, $0 < i \leq 320$

Step num-ber	Low-frequency ordered pairs: voltage (V) vs. time (s)							
	ΔV = constant		Δt = constant		ΔQ = constant		ΔE = constant	
	V_i (V)	T_V (s)	V_t (V)	t_i (s)	V_Q (V)	t_Q (s)	V_E (V)	t_E (s)
10	3.15	10.5	2.92962	2225	2.91761	2268	2.91761	2193
30	3.05	413	2.78998	6675	2.79367	6710	2.79367	6648
50	2.95	1820	2.69811	11125	2.70742	11154	2.70742	11103
70	2.85	4529	2.62297	15575	2.63477	15599	2.63477	15557
90	2.75	8608	2.55634	20025	2.56906	20044	2.56906	20011
110	2.65	13971	2.49457	24475	2.50723	24489	2.50723	24466
130	2.55	20403	2.43555	28925	2.44747	28934	2.44747	28920
150	2.45	27595	2.37788	33375	2.38853	33379	2.38853	33376
170	2.35	35189	2.32042	37825	2.32938	37823	2.32938	37831
190	2.25	42809	2.26216	42275	2.26905	42268	2.26905	42288
210	2.15	50081	2.20198	46725	2.20648	46711	2.20648	46746
230	2.05	56662	2.13853	51175	2.14034	51154	2.14034	51205
250	1.95	62252	2.06986	55625	2.06871	55595	2.06871	55666
270	1.85	66609	1.99265	60076	1.98840	60035	1.98840	60130
290	1.75	69565	1.89978	64526	1.89283	64472	1.89283	64600
310	1.65	71027	1.76600	68976	1.76142	68901	1.76142	69085
320	1.60	71201	1.60000	71201	1.60000	71201	1.60000	71201

See curves in Fig. 4.

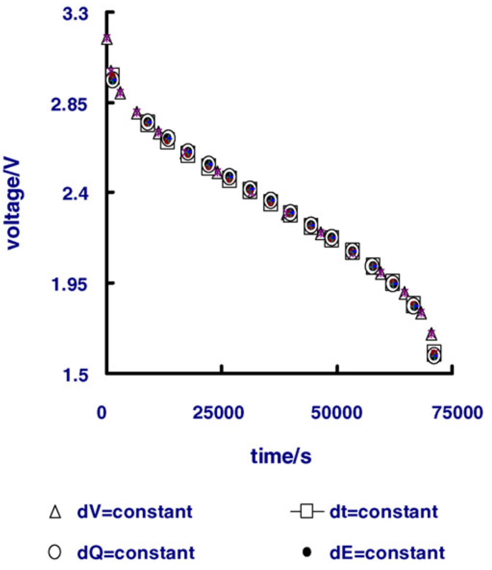

Fig. 4. Battery voltage vs. time, MALLORY battery 9K62, constant resistive load 50 Ω. Symbols: generated ordered pairs—dV = constant: $V_{\Delta V=\text{const},i}$ vs. $t_{\Delta V=\text{const},i}$, d$t$ = constant: $V_{\Delta V=\text{const},i}$ vs. $t_{\Delta V=\text{const},i}$, d$Q$ = constant: $V_{\Delta Q=\text{const},i}$ vs. $t_{\Delta Q=\text{const},i}$, d$E$ = constant: $V_{\Delta E=\text{const},i}$ vs. $t_{\Delta E=\text{const},i}$, where i: 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 340.

4. Conclusions

(1) Battery testing by calculated discharge-curve method was described. The new approach to the both battery testing procedure and data analyzing were evaluated.

(2) The procedure of a battery discharge curves generation using low-frequency monitored data by calculated discharge-curve algorithm was presented.

(3) Discharge curves, i.e. the ordered six-tuple: time–current–voltage–capacity–energy on load–internal energy losses are lying in the energy volume: initial current × initial open-circuit voltage × time, which is the regular parallelepiped.

(4) Battery's average cell was defined as the battery represented cell to which each cell of battery string may be compared.

(5) Battery/cell average current, voltage power on load and power of battery/cell internal resistance was introduced. The irregular energy volumes, as the parts of the regular parallelepiped, should be transformed into regular figures using battery/cell average characteristics.

(6) Battery/cell voltage operator was defined to which battery/cell y-operator is equal. The set of battery/cell characteristics: time, capacity, energy on load and internal energy losses, may represent y-operator. The further analyzes of y-operator varieties need to be evaluated.

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